# Chirality and Particle Models on D-brane worlds

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## STRING PHENOMENOLOGY

Link between String Theory and Particle Physics

Or: How is **Standard Model** embedded in **String Theory** ?

Basic requirements:

- Chiral $D = 4$ theory
- (MS) Standard Model gauge group (or extensions)
- 3 Fermionic generations
- etc.

Table 1: The Chiral (Super)fields of the (MS)SM.

| (Super)field | SU(3) × SU(2) × U(1) | Particles |
|---|---|---|
| $Q$ | (3, 2, 1/6) | quarks $(u, d)$ and squarks $(\tilde{u}, \tilde{d})$ |
| $\bar{U}$ | $(\bar{3}, 1, -2/3)$ | quarks $(\bar{u})$ and squarks $(\tilde{\bar{u}})$ |
| $\bar{D}$ | $(\bar{3}, 1, 1/3)$ | quarks $(\bar{d})$ and squarks $(\tilde{\bar{d}})$ |
| $L$ | $(1, 2, -1/2)$ | leptons $(\nu, e)$ and sleptons $(\tilde{\nu}, \tilde{e})$ |
| $\bar{E}$ | (1, 1, 1) | electron $(\bar{e})$ and selectron $(\tilde{\bar{e}})$ |
| $H_1$ | $(1, 2, -1/2)$ | Higgs $(h_1)$ and Higgsinos $(\tilde{H}_1)$ |
| $(H_2)$ | (1, 2, 1/2) | Higgs $(h_2)$ and Higgsinos $(\tilde{H}_2)$ |

The right charges for the theory to be

ANOMALY FREE

$< 1995$

Top-Bottom Scenario

$D = 10$ HETEROTIC STRING

$\downarrow$

$D = 4$

Intensive work: CY, Orbifolds, Gepner's Models, Free Fermions

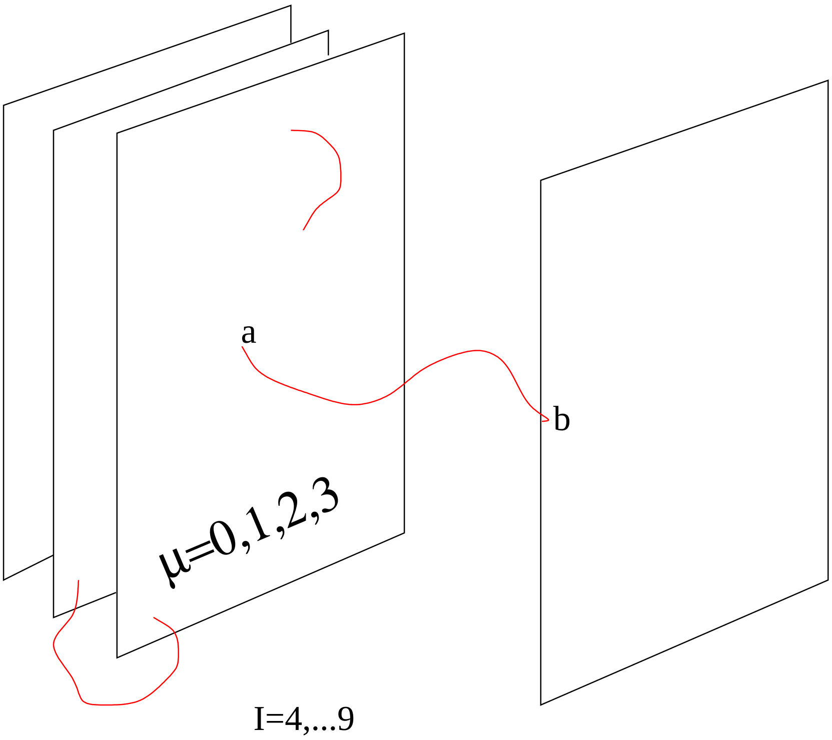
$\downarrow$

(MS) SM+ EXTRA MATTER (Doublets, triplets,. . . )

$\geq 1995$

Gauge interactions localized on extended Dp-branes

# $Brane - Worlds$




$$X^\mu(\sigma,t),\psi^\mu(\sigma,t) \qquad \mu=0,1,2,3$$
$$X^I(\sigma,t),\psi^I(\sigma,t) \qquad I=4,5,6,7,8,9$$
$$X^I(0,t)=x_a \qquad X^I(\pi,t)=x_b$$

STATES:

$$|\Psi,ab>\Lambda_{ab}$$

MASS:

$$\alpha' M^2 = \frac{y^2}{4\pi\alpha'} + N_{osc} - \frac{1}{2}$$

if

$$N_{osc} = \frac{1}{2}, \quad y = 0$$

MASSLESS STATES:

$$|\Psi^{\mu}_{-\frac{1}{2}}, ab > \Lambda^0_{ab} \quad \text{Vectors}, \quad |s_0, s_1, s_2, s_3 > \Lambda^0_{ab} \quad \text{Gaugini}$$

$$|\Psi^{I}_{-\frac{1}{2}}, ab > \Lambda^I_{ab} \quad \text{Scalars}, \quad |s_0, s_1, s_2, s_3 > \Lambda^I_{ab} \quad \text{Fermions}$$

$s_i = \pm\frac{1}{2}$

$$[U(N_1) \quad \mathcal{N} = 4] \times [U(N_2) \quad \mathcal{N} = 4]$$

**NON- CHIRAL**

## Chirality and Gauge Group

$|\Psi^{\mu}_{-\frac{1}{2}}, ab > \Lambda^0_{ab}$ Vectors, $|s_0, s_1, s_2, s_3 > \Lambda^0_{ab}$ Gaugini

with $s_{st}, s_i = \pm\frac{1}{2}$ and $\sum_i s_i = \text{odd} \begin{cases} (-\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}) \\ (-\frac{1}{2}, \underline{-\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}}) \end{cases}$

$\Lambda^0$ $n \times n$ hermitian

n x n̄ n  n̄

$$\lambda^0 \equiv E_P \quad P = (\underline{+, -, \ldots}) \quad \text{charged}$$
$$\lambda^0 \equiv H_I \quad I = 1, \ldots, n \quad \text{Cartans}$$

$\downarrow$

$\mathcal{N} = 4$ Susy $U(n)$

Some spinors must be projected out

# USUAL SCENARIOS:

## Toroidal like internal space

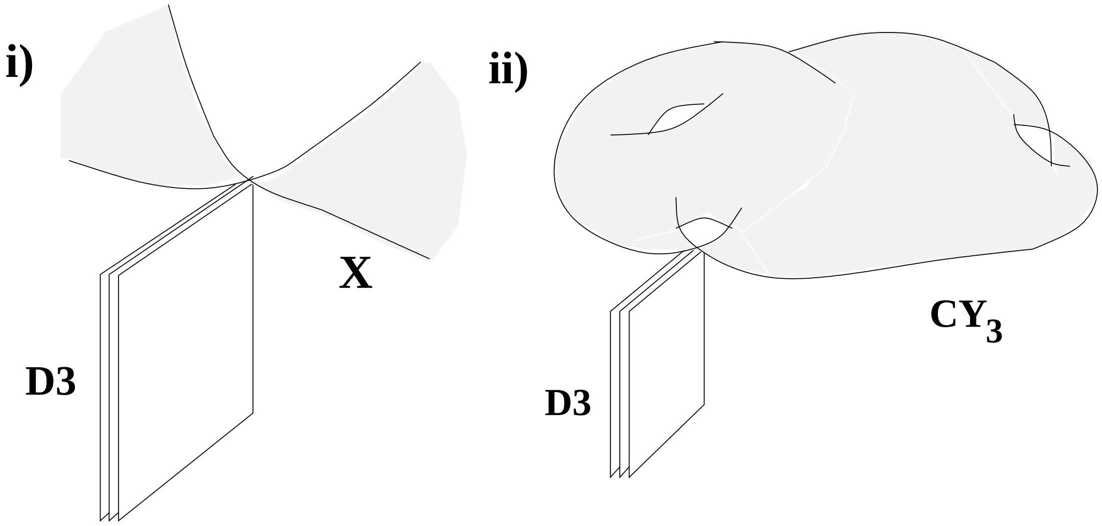
↓

Branes at orbifold singularities

Intersecting branes

Also: Branes in non trivial compactifying manifolds **i.e** **Gepner models**

## Branes at singularities




at $C^3/\mathbf{Z_M}$ singularity the orbifold action $g_M = (\theta, \gamma) \equiv (v_i, V)$

$$|\Psi > \Lambda = |\theta\Psi > \gamma^{-1}\Lambda\gamma$$

Thus, for the gauge sector

$$|\Psi^{\mu}_{-\frac{1}{2}} > \Lambda^0 = |\Psi^{\mu}_{-\frac{1}{2}}\rangle\gamma^{-1}\Lambda^0\gamma$$
$$|s_{st}, s_1, s_2, s_3\rangle\Lambda^0 = \mathrm{e}^{2i\pi v\cdot s}|s_{st}, s_1, s_2, s_3,\rangle\gamma\Lambda^0\gamma^{-1}$$

## Invariance condition

$$\Lambda^0 = \gamma\Lambda^0\gamma^{-1} \equiv P.V \quad = \quad 0 \quad \text{mod } integer$$

$U(n)$ is broken. Moreover

$$s_i v_i \quad = \quad 0 \quad \text{mod } integer$$

projects out some fermions.

### i.e.

Start with, 5 D- branes on top of each other at $C^3/\mathbf{Z_3}$ singularity

$$\begin{aligned} P &= \underline{(+,-,0,0,0)} \\ v &= (0,\frac{1}{3},\frac{1}{3},\frac{1}{3}) \\ V &= \frac{1}{3}(111,0,0) \end{aligned}$$

$$P.V = 0 \qquad U(5) \rightarrow U(3)\times U(2)$$

and only

$$|s_{st},s_1,s_2,s_3\rangle = |\frac{1}{2},-\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\rangle$$

is allowed.

$\mathcal{N}=1$ Susy $SU(3)\times SU(2)\times U(1)^2$

**CHIRAL!**

## D3-branes at a $\mathbb{R}^6/\mathbf{Z_N}$ singularity

$$g \in \mathbf{Z_N} \begin{cases} \frac{1}{N}(a_0, a_1, a_2, a_3) & on \quad \text{spinors} \\ \frac{1}{N}(0, b_1, b_2, b_3) & on \quad \text{NS} \\ \gamma_{\theta,3} \equiv V_3 & on \quad \text{Chan}-\text{Paton} \end{cases}$$

$a_0 + a_1 + a_2 + a_3 = 0 \text{ mod } N$, $b_1 = a_1 + a_2$, $b_2 = a_0 + a_2$, $b_3 = a_0 + a_1$

$$\gamma_{\theta,3} \quad \equiv \quad \mathbf{V}_3 = \frac{1}{N}(0^{n_0}, 1^{n_1}, \ldots, (N-1)^{n_{N-1}})$$

**Invariance condition** $\rho_3 {\cdot} V_3 = \begin{cases} \frac{-a_\alpha}{N} \text{ mod } \mathbf{Z} & on \quad \text{spinors} \\ \frac{b_r}{N} \text{ mod } \mathbf{Z} & on \quad \text{bosons} \end{cases}$

**33 massless spectrum**

$$\begin{array}{rc} \text{Vectors} & \prod_{i=0}^{N-1} U(n_i) \\ \text{Scalars} & \sum_{r=1}^{3} \sum_{i=0}^{N-1} (n_i, \overline{n}_{i-b_r}) \\ \text{Fermions} & \sum_{\alpha=1}^{4} \sum_{i=0}^{N-1} (n_i, \overline{n}_{i+a_\alpha}) \end{array}$$

**37+73 massless spectrum**

$$\begin{array}{ll} \text{Fermions} & \sum_{i=0}^{N-1} [\, (n_i, \overline{u}_{i+\frac{1}{2}b_3}) + (u_i, \overline{n}_{i+\frac{1}{2}b_3}) \,] \\ \text{Scalars} & \sum_{i=0}^{N-1} [\, (n_i, \overline{u}_{i-\frac{1}{2}(b_1+b_2)}) + (u_i, \overline{n}_{i-\frac{1}{2}(b_1+b_2)}) \,] \end{array}$$

D7-branes are generically required in order to achieve cancellation of RR charges.

# STANDARD MODEL at a $\mathbb{R}^6/\mathbf{Z_3}$ singularity

$$g \in \mathbf{Z_3} \left\{ \begin{array}{c} a_\alpha = v_\alpha = \frac{1}{3}(0,1,1,-2) \\ \gamma_{\theta,3} \equiv V_3 = \frac{1}{3}(0,0,0;1,1;2) \end{array} \right.$$

33 Gauge Group

$$SU(3)\times SU(2)\times U(1)_Y \times U1)^2$$

33+ 37 matter

$$3(3,2\frac{1}{6}) + 3(\bar{3},1,\frac{-2}{3}) + 3(\bar{3},1,\frac{1}{3}) + 3(1,2,\frac{-1}{2}) + 3(\bar{3},1,1)$$
$$+6(1,2,\frac{1}{2}) + 3[(\bar{3},1,-\frac{1}{3}) + (3,1,\frac{1}{3})]$$

with

$$Y = -\left(\frac{1}{3}Q_3 + \frac{1}{2}Q_2 + Q_1\right)$$

## S.M +Extra Matter

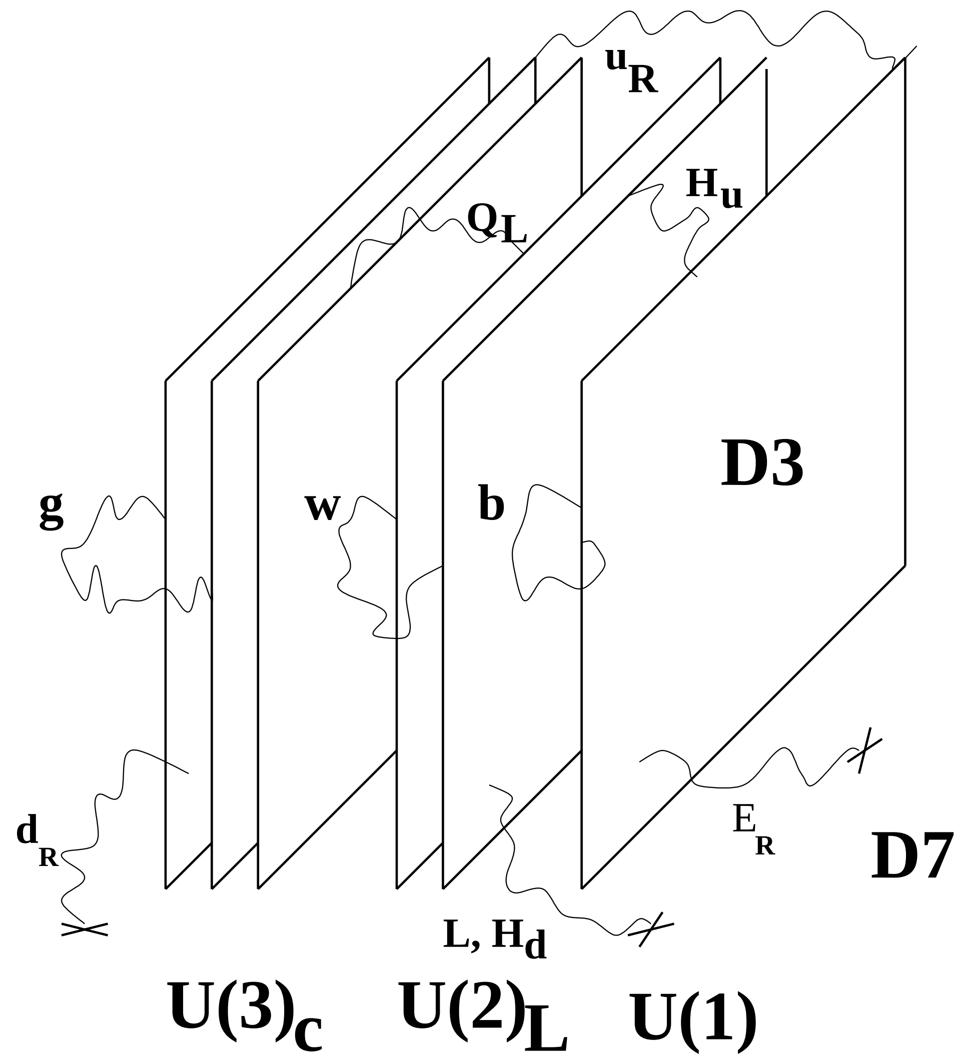



Figure 1: Six D3-branes (with worldvolume spanning Minkowski space) are located on a $\mathbf{Z_3}$. Open strings starting and ending on the same sets of D3-branes give rise to gauge bosons; those starting in one set and ending on different sets originate the left-handed quarks, right-handed U-quarks and one set of Higgs fields. Leptons, and right-handed D-quarks correspond to open strings starting on some D3-branes and ending on the D7-branes (with world-volume filling the whole figure).

IS IT POSSIBLE TO OBTAIN

# JUST THE STANDARD MODEL ?

Recall

$$U(n) \times U(m)$$
$$(n, \bar{m})$$

Hence, if we managed to have

$$3(3, \bar{2})_{(1,-1)} \quad \text{Left} \quad \text{Quarks} \quad (a_0 = a_1 = a_2 \neq a_3)$$
$$\downarrow$$
$$9(1,2) = 3(1,2) + 6(1,2)$$

6 EXTRA DOUBLETS always required by TADPOLE (ANOMALY) CANCELLATION

# WAY OUT $\quad 3 = 2 + 1$

$$2(3, \bar{2})_{(1,-1)} + (3, 2)_{(1,1)}$$

$$\text{NEED} \quad (n, \bar{m})_{(1,-1)} \quad \text{AND} \quad (n, m)_{(1,1)}$$

# ORIENTIFOLDS

# ORIENTIFOLDS

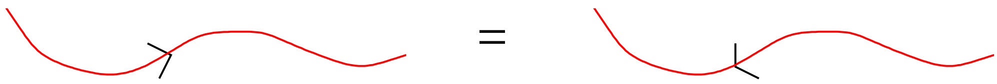
Divide by L-R symmetry in Type IIB

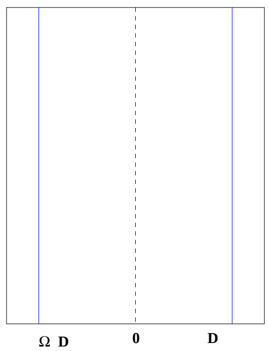
The open string end points transform as $(a, b) \rightarrow (b, a)$. An unoriented string theory is generated if strings related by $\Omega$ are identified.

Closed sector:

$$|\alpha >_L |\beta >_R \equiv \Omega(|\alpha >_L |\beta >_R) = \qquad |\beta >_L |\alpha >_R$$

Open sector: $|ab > (a, b) \rightarrow (b, a) \rightarrow$ Unoriented strings.






D-branes and their $\Omega$ images, with respect to a fixed Orientifold plane, must be included.

$$\begin{aligned} \Lambda &= -\gamma_\Omega \Lambda^T \gamma_\Omega^{-1} \\ \rightarrow \Lambda &= \pm \Lambda^T \end{aligned}$$

Symmetric, antisymmetric, $(n, m)$ representations $SO(n), Sp(m)$ groups.

JUST THE STANDARD MODEL at an orbifold-orientifold singularity?

$$\begin{aligned} P_{33} &= (\underline{\pm 1, \pm 1, 0, \ldots, 0}) \\ \mathbf{V}_3 &= \frac{1}{N}(0^{n_0}, 1^{n_1}, \ldots, (N-1)^{n_{N-1}}) \end{aligned}$$

$$\rho_3 \cdot V_3 = \frac{-a_\alpha}{N} \bmod \mathbf{Z}$$

$$3 = 2 + 1 \rightarrow a_\alpha = \frac{1}{N}(a, a, b, c = -2a - b)$$

since

$$(3, \bar{2}) \equiv (\underline{1, 0, 0}; \underline{-1, 0}; \ldots) \quad (3, 2) \equiv (\underline{1, 0, 0}; \underline{1, 0}; \ldots)$$

$$\downarrow$$

$$\mathbf{V}_3 = \frac{1}{N}(-\frac{a+b}{2}, -\frac{a+b}{2}, -\frac{a+b}{2}; \frac{a-b}{2}, \frac{a-b}{2}, d_1, \ldots)$$

33 sector

$$SU(3) \times SU(2) \times U(1)_Y \times U1)^2$$

$$2(3, \bar{2}, \frac{1}{6}) + (3, 2, \frac{1}{6}) + \ldots$$

with

$$Y = -\frac{1}{6} Q_3 + 0 Q_2 + \ldots$$

SUSY

$$b = -2a \qquad a_\alpha = \frac{1}{N}(a, a, -2a, 0) = v \to 2(3, \bar{2}, \frac{1}{6})$$

$$\downarrow$$

NON-SUSY

Also

$$a \neq -b, -c - a \quad \text{mod N} \quad \text{to} \quad \text{avoid} \quad (\bar{3}, 2)$$

$$\frac{a-b}{2} + d_i \neq -a_\alpha \quad \text{mod N} \quad \text{to} \quad \text{avoid} \quad (1, \bar{2})$$

Summary:

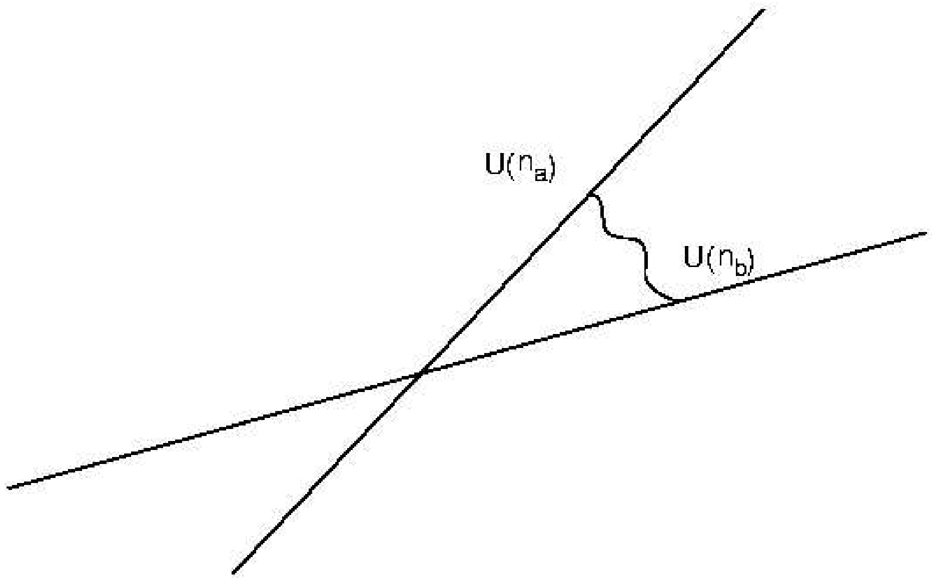
JUST THE STANDARD MODEL at an orbifold-orientifold singularity requires

NON-SUSY

$N \geq 11$

$N = 12$ (incomplete ) cristalographic example

## Intersecting branes




Chiral fermions appear at intersections.

$$\prod_{a=1}^{K} U(N_a)$$

$$\sum_{a<b} I_{ab}\,(N_a, \overline{N}_b)$$

$$I_{ab} = [\Pi_a] \cdot [\Pi_b] = \prod_i (n_a^i m_b^i - m_a^i n_b^i)$$

counts the number of times that branes in stack $a$ intersect branes in stack $b$. This number is not necessarily one (or zero) if branes wrap around cycles on compact dimensions.

Computation of string modes and quantization proceeds as in previous cases. The difference arises in the boundary conditions.

Number of families $\equiv$ the number of times that cycles intersect in the compact manifold.

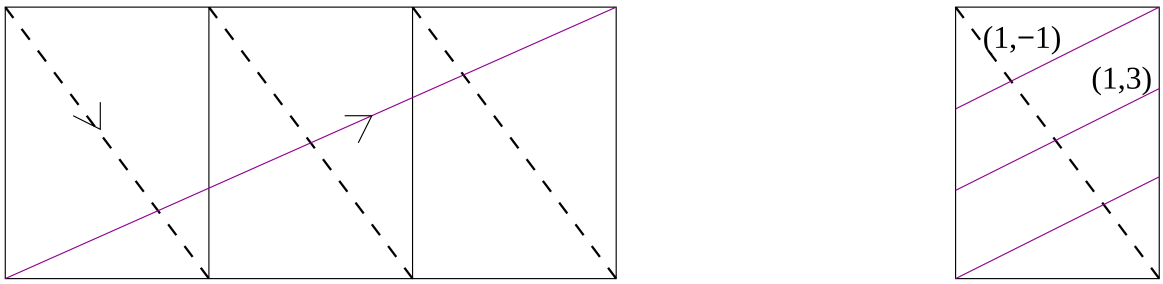



Figure 2: Two stacks of branes intersect on a $T^2$ torus. First stack wraps once on each torus cycle (one with negative orientation), denoted by $(n_a, m_a) = (1, -1)$ while the second one wraps once on $e_1$, vertical direction and three times on horizontal direction $e_2$, denoted by $(n_b, m_b) = (1, 3)$. Intersection number is thus $I_{12} = 4$

## Explicit D-brane example

### Wrapping numbers

| $stacks$ | $T_1^2$ | $T_2^2$ | $T_3^2$ |
|---|---|---|---|
| $N_1 = 3$ | (1,0) | (1,-1) | (1,-1) |
| $N_2 = 2$ | (2,1) | (1,2) | (1,0) |
| $N_3 = 1$ | (2,1) | (-1,-2) | (1,0) |
| $N_4 = 1$ | (1,0) | (1,-1) | (-2,2) |
| $N_5 = 1$ | (1,0) | (1,-1) | (-1,1) |
| $N_6 = 1$ | (2,1) | (-1,-2) | (1,0) |

$$
\begin{aligned}
I_{12} &= 3 = I_{56} = I_{25} \\
I_{13} &= I_{16} = I_{35} = -3 \\
I_{24} &= I_{46} = 6 = -I_{34}
\end{aligned}
$$

$$SU(3) \times SU(2) \times U(1)_Y \,(\times U(1)'s)$$

chiral fermion spectrum

$$
\begin{aligned}
&3(3,2,1/6)_{(1,-1,0,0,0,0)} + 3(\bar{3},1,-2/3)_{(-1,0,1,0,0,0)} + \\
&3(\bar{3},1,1/3)_{(-1,0,0,0,0,1)} \\
&+3(1,2,1/2)_{(0,1,0,0,-1,0)} + 3(1,2,-1/2)_{(0,1,0,\underline{-1,0},0,0)} + \\
&3(1,1,1)_{(0,0,0,\underline{1,0},0,-1)} + 3(1,1,-1)_{(0,0,-1,0,1,0)} + \\
&3(1,1,0)_{(0,0,0,0,1,-1)} + 3(1,1,0)_{(0,0,-1,\underline{1,0},0,0)}
\end{aligned}
$$

$$Y = -\left(\frac{Q_1}{3} + \frac{Q_2}{2} + Q_5 + Q_6\right)$$

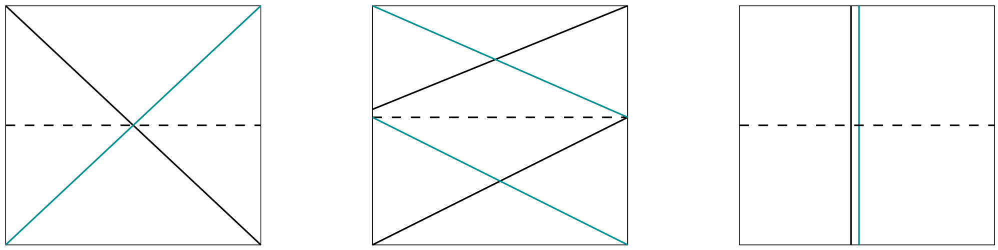
where $Q_a$ is the $U(1)$ generator in $U(N_a)$.

JUST THE STANDARD MODEL at Intersecting branes.

ORIENTIFOLDS in order to have $3 = 2 + 1$

Orientifold consistency

D6: $(n, m) \leftrightarrow \Omega\mathcal{R}$D$_a$6 :$(n, -m)$



The dashed line represent the direction where the O6-plane lives

Strings propagate between branes and their images.

The spectrum contains invariant combinations under $\Omega\mathcal{R}$.

- $D_a - D_a$: $\leftrightarrow \Omega\mathcal{R}D_a - \Omega\mathcal{R}D_a$ sector.

  This sector contains, as in the toroidal case, d=4 $\mathcal{N} = 4$ super Yang-Mills. When one brane is its own orientifold image $SO(N)$ and $USp(N)$ groups can appear

- $D_a - D_b \leftrightarrow \Omega\mathcal{R}D_b - \Omega\mathcal{R}D_a$ sector. $(N_a, \bar{N}_b)$

$$I_{ab} \; = \; (n_a^1 m_b^1 - m_a^1 n_b^1)(n_a^2 m_b^2 - m_a^2 n_b^2)(n_a^3 m_b^3 - m_a^3 n_b^3)$$

- $D_a - \Omega\mathcal{R}D_b$:$\leftrightarrow D_b - \Omega\mathcal{R}D_a$ chiral fermions in the $(N_a, N_b)$

$$I_{ab^*} \; = \; -(n_a^1 m_b^1 + m_a^1 n_b^1)(n_a^2 m_b^2 + m_a^2 n_b^2)(n_a^3 m_b^3 + m_a^3 n_b^3) \; .$$

- $D_a - \Omega\mathcal{R}D_a$: $\leftrightarrow D_a - \Omega\mathcal{R}D_a$ invariant. Fermions in the antisymmetric ,symmetric representations.

## JUST THE STANDARD MODEL example.

$$\begin{aligned} I_{ab} &= 1\,;\; I_{ab*} \;=\; 2 \\ I_{ac} &= -3\,;\; I_{ac*} \;=\; -3 \\ I_{bd} &= 0\,;\; I_{bd*} \;=\; -3 \\ I_{cd} &= -3\,;\; I_{cd*} \;=\; 3 \end{aligned} \tag{1}$$

all other intersections vanishing.

| Intersection | Matter fields | | $Q_a$ | $Q_b$ | $Q_c$ | $Q_d$ | Y |
|---|---|---|---|---|---|---|---|
| (ab) | $Q_L$ | $(3,2)$ | 1 | -1 | 0 | 0 | 1/6 |
| (ab*) | $q_L$ | $2(3,2)$ | 1 | 1 | 0 | 0 | 1/6 |
| (ac) | $U_R$ | $3(\bar{3},1)$ | -1 | 0 | 1 | 0 | -2/3 |
| (ac*) | $D_R$ | $3(\bar{3},1)$ | -1 | 0 | -1 | 0 | 1/3 |
| (bd*) | $L$ | $3(1,2)$ | 0 | -1 | 0 | -1 | -1/2 |
| (cd) | $E_R$ | $3(1,1)$ | 0 | 0 | -1 | 1 | 1 |
| (cd*) | $N_R$ | $3(1,1)$ | 0 | 0 | 1 | 1 | 0 |

Table 2: Standard model spectrum and $U(1)$ charges

$Q_a$ is $3B$, $B$ being the baryon number $Q_d$ is nothing but (minus)lepton number etc.

$$Q_Y \;=\; \frac{1}{6}Q_a \;-\; \frac{1}{2}Q_c \;+\; \frac{1}{2}Q_d$$

## D-branes on Non-toroidal manifolds

Branes wrapping cycles and intersecting on generic Calaby-Yau.

First step: Exactly solvable CFT $\longrightarrow$ i.e. Gepner Models.

$$\mathcal{Z}_T(\tau,\bar{\tau}) = \sum_{a,b} \chi_a(\tau)\mathcal{N}^{ab}\bar{\chi}_b(\bar{\tau})$$

$\chi_a(\tau) = \mathrm{Tr}_{\mathcal{H}a} q^{L_0 - \frac{c}{24}}$, with $q = e^{2i\pi\tau}$

$a \equiv (l_i, m_i, s_i)$ in Gepner,s models.

- Modular invariant $(S,T)$
- $c = 15$
- L-R symmetric $\mathcal{N}^{ab} = \mathcal{N}^{ba}$

$$\downarrow$$

Project onto Left-Right symmetric states $\frac{1}{2}(1+\Omega)$

$$\mathcal{Z}_\Omega(\tau,\bar{\tau}) = \frac{1}{2}\mathcal{Z}_T(\tau,\bar{\tau}) + \mathcal{Z}_K(\tau-\bar{\tau}).$$

$$e^{2i\pi\tau L_0} e^{-2i\pi\bar{\tau}\bar{L}_0} \longrightarrow e^{2i\pi 2it_K L_0}$$

with $\tau - \bar{\tau} = 2it_K$ and thus

$$\mathcal{Z}_K(2it_K) = \frac{1}{2}\sum_a \mathcal{K}^a \chi_a(2it_K) \quad \mathrm{Klein-Bottle}$$

where $|\mathcal{K}^a| = \mathcal{N}^{aa}$

The Klein bottle amplitude in the *transverse channel* is obtained by performing an S modular transformation such that

$$\tilde{\mathcal{Z}}_K(il) = \frac{1}{2}\sum_a O_a^2 \chi_a(il) = \langle C|e^{2\pi l H_{closed}}|C\rangle$$

with $l = \frac{1}{2t_K}$ and

$$\begin{aligned} O_a^2 &= 2^D \mathcal{K}^b S_{ba} \\ |C\rangle &= \sum_a O_a |a\rangle\rangle \quad \text{Crosscup state} \\ \langle\langle b|e^{2\pi l H_{closed}}||a\rangle\rangle &= \delta_{a,b}\chi_a(il) \end{aligned}$$

when $l \to \infty$

$$\tilde{\mathcal{Z}}_K(il) \to \sum_a O_a^2 \frac{1}{{m_a}^2}$$

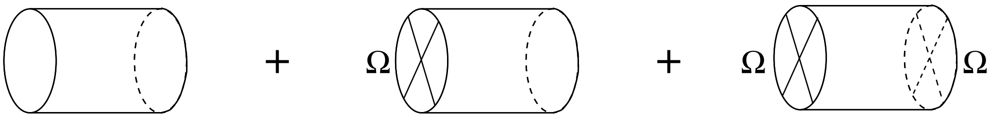
Tadpole-like divergencies $\equiv$ unbalanced RR-charge

$\downarrow$

Open strings, ending on D-branes must be added

$$\tilde{\mathcal{Z}}_C(il) = \frac{1}{2}\sum_a (D_a^\alpha n_\alpha)^2 \chi_a(l) =, n_\alpha n_\beta \langle \alpha | e^{2\pi l H_{closed}} | \beta \rangle =$$

$$|\alpha\rangle = \sum_a D_a^\alpha |a\rangle\rangle \quad \text{Boundary} \qquad \text{state}$$




$$\tilde{\mathcal{Z}}_K(il) + \tilde{\mathcal{Z}}_M(il) + \tilde{\mathcal{Z}}_C(il) \to$$
$$\sum_a (D_a^\alpha n_\alpha)^2 + 2O_a D_a^\alpha n_\alpha + O_a^2) \frac{1}{{m_a}^2}$$
$$= \quad \sum_a (O_a + D_a^\alpha n_\alpha)^2 \frac{1}{{m_a}^2}$$

Tadpole cancellation

$$\downarrow$$

$$O_a + D_a^\alpha n_\alpha = 0$$

Partition functions in direct channel $\longrightarrow$ spectrum

$$(D_a^{\alpha} n_{\alpha})^2 = C_{\alpha\beta b} n_{\alpha} n_{\beta} S_{ba}$$
$$O_a(D_{ja} n_j) = 2^{\frac{D}{2}} M_{jb} n_j P_{ba}$$

By $q$ expanding cylinder $+$ Möbius strip amplitudes,

$$\frac{1}{2}[(C_{jka} n_j n_k) \pm (M_{ja} n_j)] q^{m_a^2}$$

multiplicities of open string fields of mass $m_a$. i.e. for vector multiplet

$$\frac{1}{2}(n_j^2 - n_j) q^0 \longrightarrow SO(n_j)$$

GEPNER'S : Diagonal invariants + Phase moddings

$$\downarrow$$

**CHIRAL SPECTRUM**

## Gepner models:

Internal sector :$\prod_{i=1}^{r}(N=2)_{k_i}$ minimal models.

$$\sum_{j=1}^{r} c_{k_j}^{int} = 9 \quad , \quad c = \frac{3k}{k+2} \quad , \quad k = 1, 2, \ldots$$

Primary fields: $|(l, q, s) > \equiv |\Delta_{l,q,s}, Q_{l,q,s} >$

$$\Delta_{l,q,s} = \frac{l(l+2) - q^2}{4(k+2)} + \frac{s^2}{8} \mod 1$$
$$Q_{l,q,s} = -\frac{q}{k+2} + \frac{s}{2} \mod 2.$$

$\downarrow$

$$\chi_{(l,q,s)}(\tau, z) = \mathrm{Tr}_{\mathcal{H}_{(l,q,s)}} \left( e^{2\pi i\tau(L_0 - \frac{c}{24})} e^{2\pi i z J_0} \right)$$

A $c = 12$ supersymmetric theory requires

$$Q_{tot} = Q_\nu + \sum_{j=1}^{r} Q_{l_j, q_j, s_j} \in 2Z + 1$$

## $3^5_D \qquad c = \frac{9}{5}$

Each term is encoded in a 5 component vector (one for each theory)$\vec{\gamma}$ taking values $0$ or 1.

Only non vanishing coefficients in $\tilde{\mathcal{Z}}_K$ are

$$\mathcal{O}_{\vec{0}} = 2^4 5^{\frac{1}{8}} \kappa^{\frac{15}{2}} \qquad ; \qquad \mathcal{O}_{\vec{1}} = 2^4 5^{\frac{1}{8}} \kappa^{-\frac{15}{2}}$$

where $\vec{0} \equiv (0,0,0,0,0)$ and $\vec{1} \equiv (1,1,1,1,1)$ and $\kappa \equiv \frac{1}{2}(1+\sqrt{5})$.

Consistency determines $\mathcal{D}_{\vec{\gamma}}$ and $\mathcal{M}_{\vec{\gamma}}$ for cylinder and MS amplitude

$$\mathcal{D}^2_{\vec{\gamma}} \quad = \quad \frac{5^{\frac{1}{4}}}{\kappa^{5/2}} \frac{\left( \sum_{\vec{\delta}} \kappa^{(\vec{\gamma}-\vec{\delta})^2} (-1)^{\vec{\gamma}.\vec{\delta}} n^{\vec{\delta}} \right)^2}{\kappa^{(\vec{\gamma})^2}}$$

and

$$\tilde{\mathcal{M}}_{\vec{\gamma}} = \tilde{\mathcal{D}}_{\vec{\gamma}} \tilde{\mathcal{O}}_{\vec{\gamma}} = -\sum_{\vec{\delta}} \sqrt[4]{5} (-1)^{(\vec{\gamma})^2} \kappa^{(\vec{\gamma}-\vec{\delta})^2} (-1)^{\vec{\gamma}.\vec{\delta}} \kappa^{\frac{5}{2}-2(\vec{\gamma})^2} n_{\vec{\delta}}.$$

Tadpole cancellation equations are thus

$$\begin{aligned} \mathcal{D}_{\vec{0}} + \mathcal{O}_{\vec{0}} &= 0 \\ \mathcal{D}_{\vec{1}} + \mathcal{O}_{\vec{1}} &= 0 \end{aligned}$$

which read,

$$\begin{aligned} N_0 + N_2 + N_3 + 2N_4 + 3N_5 &= 12 \\ N_1 + N_2 + 2N_3 + 3N_4 + 5N_5 &= 20 \end{aligned}$$

where $N_i = \sum_{\vec{\gamma}\ /\ |\vec{\gamma}|=i} n_{\vec{\gamma}}$ ($e.g.$, $N_4 = n_{\underline{(1,1,1,1,0)}}$).

For instance, with

$$N_0 = n_{(0,0,0,0,0)} \quad N_1 = n_{(1,0,0,0,0)} \quad N_2 = n_{(1,1,0,0,0)}$$

Massless spectrum is found

| Internal | mult. | irrep. |
|---|---|---|
| $(0,0)^5$ | 1 | $SO(N_0) \times SO(N_1) \times SO(N_2)$ |
| $(2,2)\underline{(3,3)(0,0)^3}$ | 4 | $(1, \Box\Box, 1) + (1, 1, \Box\Box) + (N_0, N_1, 1)$ |
| $(3,3)(2,2)(0,0)^3$ | 1 | $(1, 1, \Box\Box) + (1, N_1, N_2)$ |
| $(0,0)(2,2)\underline{(0,0)^2(3,3)}$ | 3 | $(1, 1, \Box\Box) + (1, N_1, N_2)$ |
| $(1,1)^2\underline{(0,0)^2(3,3)}$ | 3 | $(1, 1, \begin{smallmatrix}\Box\\\Box\end{smallmatrix}) + (N_0, 1, N_2) + (1, N_1, N_2)$ |

with

$$N_0 = 12 - N \quad ; \quad N_1 = 20 - N \quad ; \quad N_2 = N$$

# Non-chiral

## Phase moddings

Phase symmetry group $G = \otimes_a Z_{M_a}$,

$$\theta \Phi_{l,q,s} \theta^{-1} = e^{-2i\pi\gamma \frac{q}{m}} \Phi_{l,q,s}$$

with $\gamma \in Z$ in each block. The full phase transformation can be encoded into an $r$ dimensional

$$\vec{\Gamma}^a = ({\gamma_1}^a, {\gamma_2}^a, \ldots, {\gamma_r}^a)$$

Moddings by such symmetries can be easily implemented by replacing $\chi_{\vec{l},\vec{q}}(\tau) \rightarrow \chi^{\Gamma}_{\vec{l},\vec{q}}(\tau)$

$$\chi^{\Gamma}_{\vec{l},\vec{q}}(\tau) = \frac{1}{M} \sum_{x,y} \chi^{\Gamma}_{\vec{l},\vec{q}}(\tau, x, y)$$

where the character in sector $(x, y)$ is

$$\chi^{\Gamma}_{\vec{l},\vec{q}}(\tau, x, y) = e^{-2i\pi x \frac{\gamma_i}{m}(q_i + \gamma_i y)} \chi_{\vec{l},\vec{q}+2\vec{\gamma}y}(\tau)$$

$\rightarrow$

Modular invariant:

$$\mathcal{Z}_T(\tau,\bar\tau) = \sum_{\vec\alpha;\vec{\bar\alpha}} \mathcal{N}_{\vec\alpha;\vec{\bar\alpha}} \chi^{\Gamma}_{\vec\alpha}(\tau,0) \chi^{\bar\Gamma}_{\vec{\bar\alpha}}{}^{*}(\bar\tau,0)$$

In particular $\bar\Gamma = 0$

$$Z_T = \sum_{\alpha} \left( \sum_{y} \delta\left( \frac{\Gamma}{m}(q+\Gamma y) \right) \chi_{\alpha+2\Gamma y} \right) \chi^{*}_{\alpha}$$

Klein Bottle amplitude is found by keeping identical right and left states. Thus $y = 0 \mod m/2$. In particular, for odd $m$, only $y = 0$ states are allowed leading to

$$Z_K = \sum_{\alpha} \delta\left( \frac{\Gamma q}{m} \right) \chi_{\alpha}(2it)$$

## Open sector

Embed $\theta$ as an action on gauge degrees of freedom

$$\begin{aligned} \hat{\theta}|\Phi_k; i, j\rangle\lambda_{ji} &= \gamma_{ii'}|\hat{\theta}\Phi_k; i', j'\rangle\gamma_{j'j}\lambda_{ji} \\ &= e^{2\pi i\delta_k}(\gamma^{-1}\lambda\gamma)_{j'i'}|\Phi_k; i', j'\rangle \end{aligned}$$

Therefore, invariance requires

$$e^{2\pi i\delta_k}\gamma^{-1}\lambda^k\gamma = \lambda^k$$

$$\delta_k = -\frac{\Gamma.q}{m}$$

$Z_M$ Chan-Paton twist in terms of Cartan generators as $\gamma = e^{2\pi i V H}$ where $V$ is a “shift” eigenvalues vector of the generic form

$$V = \frac{1}{M}(0^{N_0}, 1^{N_1}, \ldots, (M-1)^{N_{M-1}})$$

(ensuring $\gamma^M = 1$) and

$$\rho_k V = \delta_k$$

where $\rho_k$ is the weight vector associated to the corresponding $\lambda^k$ representation.

## Tadpoles

$$\tilde{Z}_C(il) = \frac{1}{M}\sum_{x=0}^{M-1}\sum_{\alpha\beta}(D_\alpha^\beta tr\gamma_{\alpha,x})^2\chi_{\beta+2\Gamma x}(il)$$

$$\downarrow$$

$$\frac{1}{M}\sum_{x=0}^{M-1}\sum_{\alpha}\{(O^\alpha + D_\beta^\alpha tr\gamma_{\beta,x})^2\chi_{\beta+2\Gamma x}(il)\}$$

Just replace

$$n_\alpha \longrightarrow tr\gamma_{\alpha,x}$$

Thus, zero charge condition for RR massless fields is easily found by requiring

$$D_a^\alpha tr\gamma_{a,x} + O^\alpha = 0$$

for characters $\chi_{\alpha+2\Gamma x}$ containing massless RR states.

## Example: $\mathbf{3^5_D}$

Start with ($N = 0$)

$$SO(12) \otimes SO(20)$$
$$4[(1, \Box\Box) + (12, 20)]$$

the multiplicity comes from possible permutations in

$$(2,2)(3,3)(0,0)^3 + ((2,2)(0,0)(3,3)(0,0)^2 +$$
$$(2,2)(0,0)^2(3,3)(0,0) + (2,2)(0,0)^3(3,3)$$

$$V = \frac{1}{5}(0^{n_0}, 1^{n_1}, 2^{n_2}; 0^{m_0}, 1^{m_1}, 2^{m_2})$$

with

$$\begin{aligned} \frac{1}{2}N_0 &= n_0 + n_1 + n_2 = 6 \\ \frac{1}{2}N_1 &= m_0 + m_1 + m_2 = 10 \end{aligned}$$

i.e.

$$\begin{aligned} \rho_{(Adj,1)} &= (\underline{\pm 1, \pm 1, 0, \ldots, 0}; 0 \ldots, 0) \\ \rho_{(1,\Box\Box)} &= (0, \ldots, 0; \pm(\underline{1, 1, 0, \ldots, 0})) + (0, \ldots, 0; \underline{\pm 2, \ldots, 0}) \\ \rho_{(12,20)} &= (\underline{\pm 1, 0, \ldots, 0}; \underline{\pm 1, 0, \ldots, 0}) \end{aligned}$$

$$N_0 \qquad \rightarrow tr\gamma_{0,x} = 2n_0 + 2n_1 cos\frac{2}{5}\pi x + n_2 cos\frac{4}{5}\pi x$$
$$N_1 \qquad \rightarrow tr\gamma_{1,x} = 2m_0 + 2m_1 cos\frac{2}{5}\pi x + 2m_2 cos\frac{4}{5}\pi x$$

Tadpole cancellation equations thus read

$$(D_a^\alpha tr\gamma_{a,x} + O^\alpha)^2 \chi_{\beta+2\Gamma x}(il) = 0$$

for all $x$ twisted states such that $\vec{\alpha} + \Gamma x$ contains a RR massless state. For the example at hand such states are:

$$\begin{aligned}
44 + 20\sqrt{5} &= \pm(2tr\gamma_{0,0} + tr\gamma_{1,0} + \sqrt{5}tr\gamma_{1,0}) \\
8 &= \pm(11tr\gamma_{0,0} + 5\sqrt{5}tr\gamma_{0,0} - 7tr\gamma_{1,0} - 3\sqrt{5}n_1) \\
12 + 4\sqrt{5} &= \pm(4tr\gamma_{0,2} + 2\sqrt{5}tr\gamma_{0,2} + 7tr\gamma_{1,2} + 3\sqrt{5}tr\gamma_{1,2}) \\
12 + 4\sqrt{5} &= \pm(4tr\gamma_{0,2} + 2\sqrt{5}tr\gamma_{0,2} - 3tr\gamma_{1,2} - \sqrt{5}tr\gamma_{1,2}) \\
16 + 8\sqrt{5} &= \pm(3tr\gamma_{0,4} + \sqrt{5}tr\gamma_{0,4} + 4tr\gamma_{1,4} + 2\sqrt{5}tr\gamma_{1,4}) \\
16 + 8\sqrt{5} &= \pm(3tr\gamma_{0,4} + \sqrt{5}tr\gamma_{0,4} - tr\gamma_{1,4} - \sqrt{5}tr\gamma_{1,4})
\end{aligned}$$

Solutions,

$$n_0 = 2 \quad n_1 = 4 \quad n_2 = 0$$
$$m_0 = 2 \quad m_1 = 4 \quad m_2 = 4$$

$$\downarrow$$

$$SO(4) \times U(4) \times SO(4) \times U(4) \times U(4)$$

with

CHIRAL matter content

$$(1,1;4,4,1) + (1,1;1,\bar{4},4) + (1,1;1,1,\bar{1}0) + (1,\bar{4};1,1,4) +$$
$$(1,4;1,\bar{4},1) + (4,1;1,4,1) + 2[1,1;1,10,1) + (1,1;1,\bar{4},\bar{4})]$$
$$+2[(1,1;4,1,4) + (1,\bar{4};1,\bar{4},1) + (4,1;1,1,4) + (1,4;4,1,1)]$$

# SUMMARY AND OUTLOOK

- Chiral $D = 4$ theory
- (MS) Standard Model gauge group (or extensions)
- 3 Fermionic generations
- etc.

Obtainable in:

$$\left\{ \begin{array}{c} \text{Branes at } \textbf{orbifold} \text{ singularities} \\ \textbf{Intersecting} \text{ branes (on generic manifolds)} \end{array} \right\}$$

**MUCH MORE HAS BEEN DONE :** Yukawa couplings, stability, moduli stabilization,...

**MUCH MORE TO BE DONE**

**D-branes at singularities:**


- G. Aldazabal, L. E. Ibáñez, F. Quevedo, A. M. Uranga, D-branes at singularities: A Bottom up approach to the string embedding of the standard model, JHEP 0008:002,2000 [hep-th/0005067].
- D. Bailin, G.V. Kraniotis, A. Love, Supersymmetric Standard Models on D-branes, Phys.Lett.B502:209-215,2001
- D. Berenstein, V. Jejjala and R.G. Leigh, The Standard Model on a D-brane, hep-ph/0105042.
- L. F. Alday and G. Aldazabal, In quest of "just" the Standard Model on D-branes at a singularity, JHEP 0205:022,2002, hep-th/0203129.

**Branes at angles:**

- R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, JHEP **0010**, 006 (2000) [arXiv:hep-th/0007024];
- C. Angelantonj, I. Antoniadis, E. Dudas, A. Sagnotti, Phys. Lett. B **489**, 223 (2000) [arXiv:hep-th/0007090];
- G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan, A. M. Uranga, JHEP **0102**, 047 (2001) [arXiv:hep-ph/0011132];
- G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan, A. M. Uranga, J. Math. Phys. **42**, 3103 (2001) [arXiv:hep-th/0011073].
- A. M. Uranga, Class. Quant. Grav. **20**, S373 (2003) [arXiv:hep-th/0301032];
- D. Lüst, Class. Quant. Grav. **21**, S1399 (2004) [arXiv:hep-th/0401156]; E. Kiritsis, Fortsch. Phys. **52**, 200 (2004) [arXiv:hep-th/0310001].
- L.E. Ibanez, F. Marchesano, R. Rabadan,JHEP 0111 (2001) 002 ( hep-th/0105155) Ralph Blumenhagen, Mirjam Cvetic, Paul Langacker, Gary Shiu, *Toward realistic intersecting D-brane models*, [hep-th/0502005]. **Branes in Non-toridal constructions:**
- C. Angelantonj, M. Bianchi, G. Pradisi, A. Sagnotti and Y. Stanev,*Comments on*

*Gepner models and Type I vacua in string theory*, Phys. Lett. **B387** (1996) 743, hep-th/960722.

- A. Recknagel and V. Schomerus, Nucl. Phys. **B531** (1998) 185, hep-th/9712186 A. Recknagel, *Permutation branes*, JHEP **0304**:041,2003; hep-th/0208119
- M. Gutperle and Y. Satoh, Nucl. Phys. **B543** (1999) 73
- Ilka Brunner, Kentaro Hori, *Notes on orientifolds of rational conformal field theories*, hep-th/0208141; *Orientifolds and mirror symmetry*, hep-th/0303135.
- S. Govindarajan, J. Majumder, *Crosscaps in Gepner Models and Type IIA Orientifolds*,hep-th/0306257.
- R. Blumenhagen,*Supersymmetric orientifolds of Gepner models* JHEP **0311**, 055 (2003) [arXiv:hep-th/0310244] R. Blumenhagen and T. Weigand,*Chiral supersymmetric Gepner model orientifolds*,JHEP **0402**, 041 (2000),[arXiv:hep-th/0401148]
- I. Brunner, K. Hori, K. Hosomichi and J. Walcher, *Orientifolds of Gepner models* hep-th/0401137
- T.P.T. Dijkstra, L. R. Huiszoon, A.N. Schellekens,*Chiral Supersymmetric Standard Model Spectra from Orientifolds of Gepner Models hep-th/0403196*
- *G.A., E. C. Andrés, M. Leston, C. Nez Type IIB orientifolds on Gepner points JHEP 0309 (2003) 067,hep-th/0307183*
- *G. A., E. C. Andrés, J. E. Juknevich Particle models from orientifolds at Gepner-orbifold points JHEP 0405 (2004) 054 hep-th/0403262*